A MARKOVIAN CROSS-IMPACT MODEL

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The author describes a cross-impact method which is based on Markov-chain theory. This allows scenarios to be derived which explicitly include the time dimension. An example of the use of the method is included which deals with the installation of nuclear reactors for energy production.

Two somewhat different courses of research have been investigated by cross-impact methods: some are attempts to construct scenarios by picking events haphazardly for the sake of practicality to users,[1] unfortunately they are not soundly backed by the theory of probabilities and give doubtful results as shown by Florentin and Dognin.[2] Others are attempts to achieve more rigour by trying to produce images that are consistent with the future at a given horizon.[3,4]

These last methods have generated a growing interest because they fundamentally allow for filtration of often contradictory information given by an expert when questioned on the probabilities of various possibilities. The SMIC 74 method is the only one to date having succeeded in issuing results that are apparently compatible with the requisites of mathematical exactness.

In a joint study by Aéroport de Paris and SEMA in which the SMIC method was applied,[5] we were prompted into seeking a way of obtaining trend scenarios in which the temporal variable appears explicitly (unlike situation scenarios). This raises questions about cause and effect, a more intuitive formulation than conditional probabilities.

The Markov chains theory provides a well-adapted theoretical framework for this problem.

The example used in this article is based on a study carried out by EDF (French National Electricity Company) and CEA (Atomic Energy Commission).[6]

Concept of the model

The concept of "event", which is the basis of all cross-impact methods must be accurately defined. An event will be defined as a permanent modification,

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able to be localised in the time period of the system under study. Thus, for example, a "300% increase in the production price of oil" is an event as we understand it.

Having defined n events, we will define the "state of the system" or, more simply, "state" the occurrence or non-occurrence of each event considered. As each event is either accomplished or not accomplished there are 2^n possible states (see the Appendix for the longer mathematical formulae).

Having defined a discrete time scale, the passage from one state to another at a given instant on the time scale will be called a transition, and we shall make the following assumption:

Assumption 1: the probability of a transition only depends on the initial state and the state following transition

This means that the probability of a transition from state $\boldsymbol{E}_i$ to state $\boldsymbol{E}_j$ at instant t depends neither on t nor on the succession of transitions having brought the system to state $\boldsymbol{E}_i$. Considering the definition we have chosen for the events, this assumption is not excessively restrictive for the planned practical applications.

The random function $\boldsymbol{E}(t)$ which we shall call a scenario, is a stationary and homogeneous Markov chain,[7] and it is consequently completely defined by its transition matrix, in other words by the 2^{2n} transition probabilities: if we can evaluate all the elements of this matrix we can calculate the probability of each scenario for each initial state, the probability of the data of appearance of each event, the most probable state at each instant, etc.

However, the number of elements in the matrix rapidly becomes very high if the number of events taken into account increases: for five events there are 1024 transition probabilities, for six events there are 4096. This number of questions presented to consulted experts would become excessive; but actually the number of nonzero elements in the matrix is much lower if we omit impossible transitions (see Appendix). The number of nonzero elements for n events then becomes:

$$\sum_{p=0}^{n} 2^{n-p}\, C_n{}^p = 3^n,$$

that is 243 for five events, or 729 for six events. The amount of data required can further be reduced through an additional assumption:

Assumption 2: The events are independent during a unit time interval

The shorter the time interval selected the more this assumption holds true; as the interdependence of the events is taken into account when passing from instant t to instant $t+1$, the approximation thus made is then acceptable.

Under this assumption, the probabilities of transition from a given state to each of the subsequent possible states can be calculated from the probabilities of each of the new possible events, and the number of questions required to determine the transition matrix for n events becomes:

$$\sum_{p=0}^{n} 2^{n-p}\, C_n{}^p = n2^{n-1},$$

ie 80 for five events, or 92 for six events.

The particular nature of each system studied can further reduce the data required when setting up the transition matrix. Until now we have implicitly postulated that each event has an influence on the occurrence of each of those following.

If, in a particular problem, the occurrence or nonoccurrence of an event e_i has no influence on the occurrence of e_j, the probability of transition from a state comprising e_i to a state comprising e_j is the same as if, in the initial state, e_i does not occur, and the corresponding data are thus already provided in another connection. It is therefore interesting to determine qualitatively the mutual influences of the foreseen events in the first place, which amount to $n(n - 1)$ for n events.

If one of these influences is considered to be zero by the experts questioned, the amount of data required is reduced by 2^{n-2} (the number of states containing e_i and not e_j), ie eight for five events or 16 for six events. No general formula can be given to determine the number of questions required, for the number depends on the structure of the qualitative influences; however, it can easily be taken into account while drawing up the questionnaire.

One last problem needs to be solved which concerns the selection of a time interval. Only if an adequate number of unit intervals is inserted in the period under consideration, so that the interaction mechanisms may work perceptibly and the assumption of independence be acceptable, will the model be attractive.

However, it is difficult for the expert questioned to give estimates of transition probabilities in a restricted interval of time. Most of the long-term forecast problems extend over 10–20 years and these experts can only give satisfactory answers on the probabilities of events occurring in a 5–10-year interval. Furthermore, in order to compare results given by two experts who have selected a different time interval, it is advisable to adjust the answers to a common time scale. To this end another assumption is made:

Assumption 3: the probability density of an event occurring during a time interval selected by the expert is constant

This assumption is the most exposed to criticism among those presented in this study, for it overlooks such phenomena as, for example, a minimum lapse of time between the occurrence of two events. It will therefore be necessary to make sure it is sound when adjusting the time interval.

Under this assumption, if the interval selected by the expert lasts k time units and if P is the probability of an event occurring during this interval, the probability of the same event during a unit interval is P′

$$P' = 1 - (1 - P)^{1/k}.$$

We may systematically resort to this stratagem if we are fundamentally concerned with the qualitative results of the model, eg when we wish to know the events having a favourable or unfavourable impact on the occurrence of a given event or a given state.

Application of the model

The application can be broken down into three phases:

- selecting the events, and qualitatively analysing their interactions;

- drawing up of the questionnaire comprising the minimum number of questions necessary to determine the transition matrix;
- processing the answers to the questionnaire.

The first phase
This is the most important phase. Tests have revealed that the prime condition for obtaining relevant results is that the events be perfectly outlined. We will strive to analyse the immediate consequences of each event, as well as the circumstances conducive to its occurrence. In this phase, we may resort to a qualitative-analysis model of the system structure studied.[8]

These models make it possible to outline the system under study and to select the events playing the most significant part in its development, while being strictly interdependent. At the end of this phase we have an accurate description of the selected events (five or six at the most so as to reduce the amount of data required from experts) and a table showing the existence or absence of an event's direct influence on the occurrence of each of the other events.

The second phase
We have a small program by which the questionnaire may be written out. It comprises the necessary questions for constructing the interaction table mentioned above. The expert fills in the questionnaire and indicates the length of time for which his probability estimates are to run.

Unlike other cross-impact methods, the coherence of the answers given cannot be tested with this model because they are very few and are therefore not redundant. It is up to the expert to check his answers himself by verifying, after filling in the questionnaire, that the probabilities ascribed to each event vary in the "right" direction (according to his own feeling about it) with reference to the first phase.

The third phase
The experts' answers are processed by a program which computes the transition matrix and, for each possible initial state, gives

- the most probable state for each instant on the time scale,
- the distribution function for the dates when each event occurs,
- the most probable development up to a given time horizon.

In a normative approach, the driving events can easily be deducted from these results for a given objective by comparing the distribution functions of an event obtained for each initial state.

Example: nuclear energy[9]

The example was set up with the participation of an expert on nuclear industry development and cross-impact method problems, for the definition of the system studied as well as for the answers to the questionnaire on probabilities.

The events

Event 1: Start of operation breeder reactors
The idea is to chart the development of the nuclear industry by a significant event in the programme presently being considered in France, and on which environmental

opposition is crystallising. This event will be the introduction of breeder power plants at the planned rate of approximately 1000 MW per year during a period of time selected by the expert.

Event 2: Fossil-fuel shortage
The event probability will help identify the risk the economy is running of being seriously affected by the lack of fossil fuels: either through the exhaustion of supplies, or by the cost of halting industrial growth.

Event 3: Halting growth in per capita energy demand
The third event is a change of industrial policy in the developed countries, directing development towards innovations aimed at reducing energy requirements in a community more concerned with quality of life than with quantity of materials assets. It is reflected in energy usage per capita.

Event 4: Law against breeder reactors
Here we are trying to assess the prospects for the development of energy production. This event is a break in the planned development of the nuclear industry; eg a very strict regulation practically banning the construction and operation of breeder reactors.

Event 5: New energy sources
Lastly, an alternative to nuclear energy may be introduced into the system in a foreseeable future, eg geothermal heat, solar energy, or any other new energy source. The substitute must be clearly determined and timed so that the event will square with the above definition, and we shall therefore consider such a technology change as modifying the structure of energy procurement extensively within a few years at the most.

Limitations

We have restricted ourselves to these five events for two reasons, first to keep the number of probability estimates requested down. It does not seem possible to reduce this number by assuming the mutual influences among events to be insignificant, because each of them could directly affect the advent of the others. This, in fact, is the advantage of such an analysis. Consequently, the expert had to provide 80 probability estimates to describe the system.

The second reason for this limitation results from an experiment with a cross-impact method made in 1972 in which 20 events had been considered. This required conditional event probability estimates, besides *a priori* probabilities of each of them. Clearly the management of some 400 interactions required such brain racking that the answers were hardly reliable, so that the process leading to *a posteriori* probabilities could only partially correct the distortions caused by the expert's weariness.

Drawing up the questionnaire

The questionnaire must produce coherent answers because our method, unlike those mentioned previously, does not systematically correct the probabilities. The programme which produces this questionnaire automatically presents the various possible states of the system sequentially and, in each of them, asks for the one, or the several, event probabilities required for the construction

TABLE 1. LAYOUT OF A QUESTIONNAIRE PAGE

QUESTIONNAIRE X-1-M	PUNCHED CARD CODE | | 7 | 0 COL 1 2
IF THE FOLLOWING EVENTS HAVE OCCURRED:	BUT NOT THE FOLLOWING EVENTS:
GROWTH INTENTIONALLY STOPPED LAW AGAINST BREEDERS UTILISATION OF NEW ENERGY SOURCES	INTRODUCTION OF BREEDERS GROWTH STOPPAGE THROUGH SHORTAGE

WHAT IS THE PROBABILITY THAT THE FOLLOWING MAY HAPPEN DURING THE SUBSEQUENT PERIOD:

INTRODUCTION OF BREEDERS	COL | | | 11 12
GROWTH STOPPAGE THROUGH SHORTAGE	COL | | | 21 22

[Reply by indicating the probability (in %) in the corresponding spaces]

of the transition matrix. The expert is placed in a clearly described situation and can thus appreciate the overall causal relations leading to his estimate. Table 1 shows one of the 31 pages of the questionnaire. The set of answers given by the expert is shown in Table 2.

A careful look at Table 2 reveals a number of peculiarities. The probability attributed to event 2 (crisis due to fuel shortage), for example, is set at 10% in state $\boldsymbol{E}_0$ (none of the other events have occurred) and at 20% is state $\boldsymbol{E}_{16}$ (when breeder reactors are being introduced).

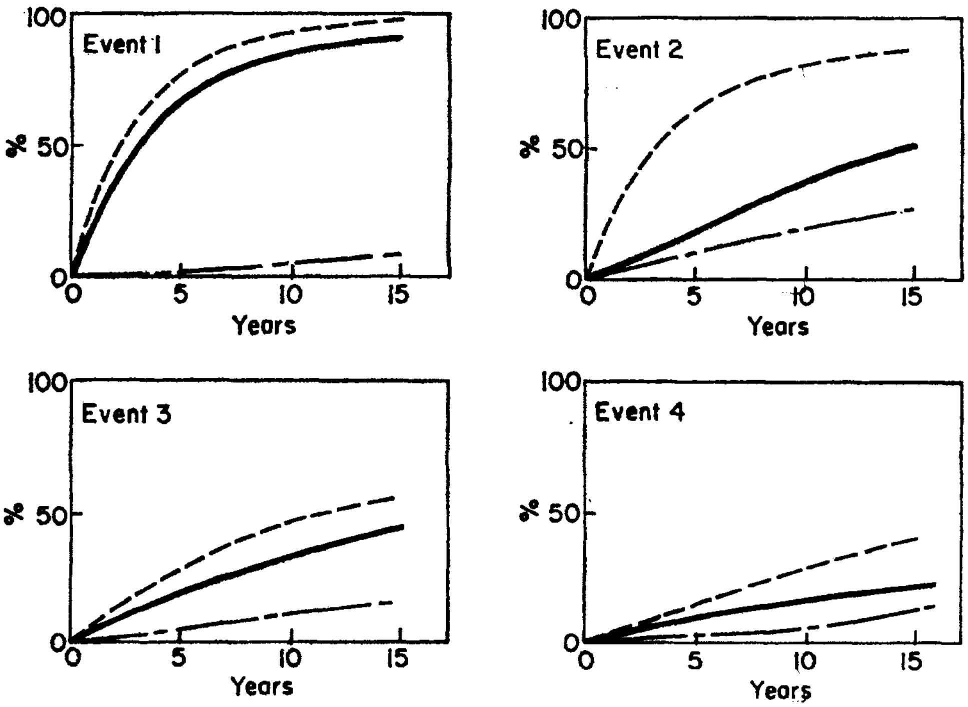
TABLE 2. ANSWERS TO THE QUESTIONNAIRE (probability of the events occurring, given the initial state)

Initial state		Probability of event (%)					Initial state		Probability of event (%)				
		1	2	3	4	5			1	2	3	4	5
0	00000	70	10	20	10	30	16	10000		20	20	5	20
1	00001	70	50	20	20		17	10001		20	10	5	
2	00010	0	70	20		50	18	10010		70	20		20
3	00011	0	20	30			19	10011		50	10		
4	00100	50	20		10	50	20	10100		20		5	30
5	00101	40	20		20		21	10101		10		10	
6	00110	0	10			50	22	10110		10			30
7	00111	0	10				23	10111		10			
8	01000	80		10	0	40	24	11000			20	0	30
9	01001	70		10	0		25	11001			10	5	
10	01010	50		10		40	26	11010			10		40
11	01011	50		10			27	11011			5		
12	01100	50			10	40	28	11100				15	35
13	01101	40			10		29	11101				15	
14	01110	40				45	30	11110					40
15	01111	20					31	11111					

This is not surprising: indeed, answers to the same question asked in different ways and at different times unavoidably undergo some diversification. Should the expert improve the coherence of his answers by comparing the probabilities given for one same event in different states after completing the questionnaire, just as we have done? This procedure may lead to the introduction of a bias in the given estimates with respect to the intuitive knowledge the expert has of the system being studied. Modification of the answers for the sake of logic should therefore be strictly limited. Fortunately, the results obtained by this method are remarkably stable *vis-à-vis* such modifications, this stability being simply due to the fact that the large number of probabilities given, as compared with the number of events studied, partially corrects this diversification, at least for certain types of results.

Results

We have mainly considered the qualitative aspect of the events' interactions and consequently the length of a period could conveniently be reduced from five years, as estimated by the expert, to one year so as to multiply the transitional stages of the process. Having computed the transition matrix **P**, dimension 32 × 32, we have then calculated its successive powers to the fifteenth, and all the results are derived from the observation of these $\mathbf{P}^k$ matrices for $k = 1$ to 15. In the subsequent part of the report a state will be marked by



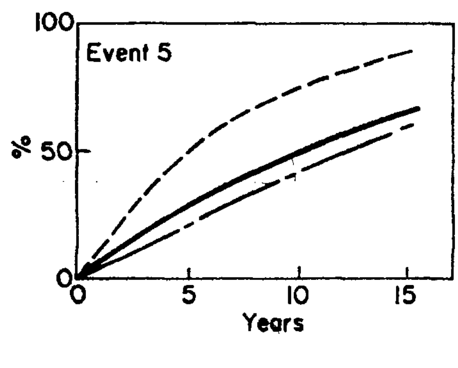


Figure 1. The distribution functions for the occurrence of each event. The broken lines give the maximum and minimum probabilities. The corresponding initial states are as follows: for event 1 an initial state of 01000 gives the maximum probability of occurrence; 00111 the minimum. Event 2: 10010 (maximum); 00110, 00111, 10110, 10111 (minimum). Event 3: 00011 (maximum; 11011 (minimum). Event 4: 00101 (maximum); 01000 (minimum). Event 5: 00110 (maximum); 10001 (minimum)

a sequence of five binary numbers (x_1, x_2, x_3, x_4, x_5) where x_i equals 0 or 1 depending on whether the corresponding event occurs or not.

The distribution function of event occurrence

The element (i, j) of the kth power of the matrix is the probability of being in state $\boldsymbol{E}_j$ after k years starting from state $\boldsymbol{E}_j$.

To obtain the probability that event e_i has occurred before k years, for a given initial state, all that has to be done is to add up the elements of the matrix $\mathbf{P}^k$ for the line corresponding to the selected initial state and for the columns corresponding to the various states containing event e_i.

Thus, starting at state $\boldsymbol{E}_0$, in which none of the selected events has occurred, we obtain the distribution functions of the dates of occurrence of each event (see Figure 1).

For example, the time required to reach a 50% probability for each event is: 3 years (event 1); 15 years (event 2); approximately 20 years (event 3); well beyond the span of the study, and perhaps never (event 4); 11 years (event 5).

The point of these results is mainly to show the variability of these numbers with the initial state. Thus, if we consider event 1 "start of operation of breeder reactors", we find that the probability of this event occurring after 10 years is: 85% starting from state 00000; 93% starting from state 01000 (oil shortage); 3% starting from state 00111 (change of social policy, law against breeders, and the use of new sources of energy). The influence of the last three events taken independently reduces the initial probability from 85% to 39% for event 4, to 66% for event 3, and to 82% for event 5.

The same analysis for event 5 "use of new energy sources", gives the following results for a 10-year period: 49% starting from state 00000; 74% starting from state 00110; 42% starting from state 10000. (The individual influences of events 3 and 4 reduce the probability to 66% and 70% respectively.)

The search for scenarios

The possibility of obtaining scenarios having a clear time scale is the most interesting aspect of this method. Indeed, one or several scenarios, in which the events taken into account are clearly outlined, are required in all long-term forecast studies whose results depend on a number of contingencies; whatever the reservations about such a description of the future, it is obvious that some rationality at this stage is better than total empiricism.

A scenario being a series of transitions between states, it seems natural to try to find out what is the most probable sequence, in order to draw up a trend scenario. Actually, this problem does not make much sense: indeed, the probability of a sequence of n states is:

$$\prod_1^n \mathrm{P}(\boldsymbol{E}_i, \boldsymbol{E}_j),$$

where $\mathrm{P}(\boldsymbol{E}_i, \boldsymbol{E}_j)$ is the probability of transition from state $\boldsymbol{E}_i$ to $\boldsymbol{E}_j$.

Now, the maximum this product may reach for a given initial state depends on n and, for different values of n, it can give series of states which differ even from the first elements. Furthermore, the probability of such a series is extremely low: for a 10-year horizon and five events, for example, there are 11^5, ie approximately 160 000 scenarios having an *a priori* probability greater than zero.

We could then think of considering the maximum probability transition at each stage, for the purpose of defining a trend scenario independent of the selected time horizon. It is easy to show that this choice if unhelpful, for if we reduce the time interval, the most probable transition will be the preservation of the initial state, and "the most probable scenario" turns out to be the one in which nothing happens.

The solution we propose is to consider the most probable sequence of states at each instant for a given initial state. The drawback is that it is not necessarily a possible path followed by the process, for we may eventually obtain two consecutive states connected by an impossible transition. On the other hand there are several advantages:

- The horizon of the study is independent.
- There is stability *vis-à-vis* variation in the unit time interval: whatever the choice of interval between transitions, the dates of occurrence of the events are generally constant.
- Generally, this sequence of states is a possible scenario (this was shown by various tests of the model).
- The dates of occurrence of the events in this sequence of states approximate the dates for which the cumulated probability of occurrence exceeds 0·5

Practically, we obtain this sequence of states by retaining the state $\boldsymbol{E}_j$ corresponding to a maximum of $P^k(i, j)$ for each value of k and for each initial state $\boldsymbol{E}_i$. In the example presented here and for the initial state 0 (Table 2),

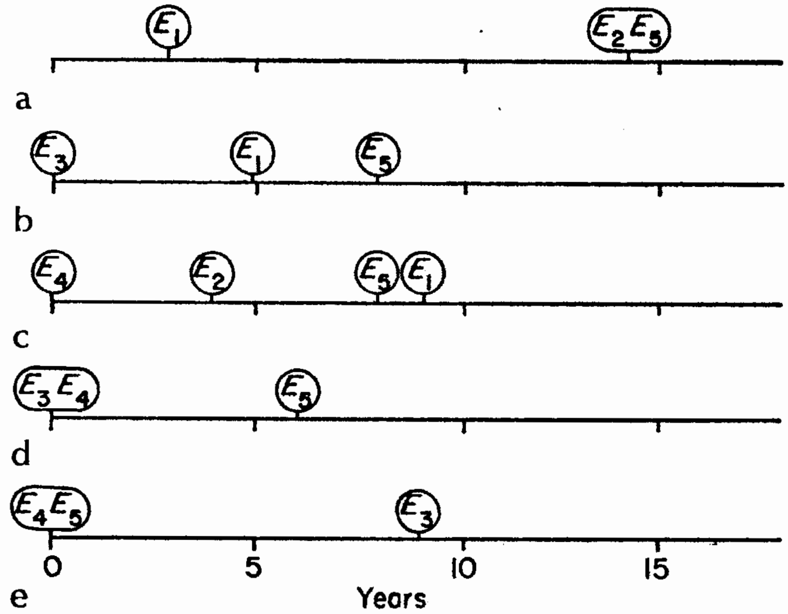


Figure 2. Trend scenarios and contrasting scenarios: (a) trend, initial state 00000; (b) initial state 00100; (c) initial state 00010; (d) initial state 00110; (e) initial state 00011

the sequence is one in which we obtain event 1 after three years, then events 2 and 5 after 11 years, ie the development of nuclear energy in the medium term. Then, in the long term and approximately simultaneously, the exhaustion of oil resources and their replacement by new sources (Figure 2).

One only has to choose a different initial state, containing ~~one~~ of the events that did not occur in the trend scenario, for instance, to obtain contrasting scenarios.

If we assume event 3 (change of industrial policy) as part of the initial state, we obtain the following scenario: event 1 occurs after five years, ie two years later than in the trend scenario, and event 5 after three years, ie six years earlier; event 2 does not occur any more. The scenario is thus of a community consuming less energy, more cautious about nuclear systems and pressing forward in research on other energy sources. This scenario is quite typical of the aspirations of a number of scientists.

Let us now take a more conflicting scenario by assuming that a law is enacted against nuclear development. We then see that event 2 occurs after four years (oil shortage), then four years later and five years later, events 5 and 1 occur respectively. The sequence of states thus obtained does not follow a permissible path because event 5 occurs at state 00011, in which event 2 has disappeared. However, the sequence is still intuitively satisfactory in so far as the shortage disappears naturally as a result of the use of all forms of energy.

This singularity actually results from a certain amount of ambiguity in the choice of the events: the fact that some of them are in opposition can hardly be reconciled with the results (ie their stability) shown by the transition matrix.

In these three scenarios, the development of the nuclear industry seems to be unavoidable. The initial conditions required to avoid it are:

- 00011: the use of new energy sources, and a law against breeders;
- 00110: a change of policy, and a law against breeders;
- 00111: a combination of the above;
- 01111: ditto, in spite of oil shortage.

This brief description of very different results obtained by simply observing the successive powers of the transition matrix reveals the richness of the method quite well, and it has not been entirely explored yet.

It would, for instance, be interesting to indicate one or several states with a sufficient probability level for each initial state, besides the most probable state at each instant; we would probably obtain a range of possibilities for each date of events in the scenarios, which would enable us to measure the stability with respect to given data.

The stability of the results

This is a crucial problem, because the method is worthwhile only if the results obtained do not depend critically on the accuracy of the probability estimates. In our example, a first test had been made with unrevised data, with no check as to the cohesion of the answers. The main peculiarities concerned probabilities related to event 2: on the average, they were markedly higher when event 1 occurred than when it did not. Furthermore, the effect of event 5 on event 4 appeared in certain reversed cases, as did the influence of event 4 on event 5. Altogether 11 values of probabilities have been modified, by exchange of given values or correction of estimates, the extent of modifications ranging from 5% (on a 10% estimate) to 50% on three values and 60% on two others. In spite of these changes, we observe two points.

First, the distribution functions are practically unchanged for all the initial states, as the variations practically never exceed 5%; two or three of them change markedly, especially that of event 2 for the initial state 0, but this could be expected as the main probability modifications concerned this event.

Second, the scenarios obtained as indicated above undergo little change, as the variations relate to time separation between them, and not to the sequence of occurrence of events. This stability has already been explained: the probability of an event or state at a given time is the sum of probabilities of a very large number of paths of the process, so that even if the latter are extensively modified, the resulting effects on the issues we want to observe are unimportant. Thus, asking for 80 estimates to study a system having only five events restores the cohesion which sometimes seems to be lacking in the answers.

Conclusion

It may look, from the above example, that the results obtained were intuitively obvious and that it was not necessary to go through this complex system.

We believe that, on the contrary, it shows that the method achieves the desired goal. Indeed, let us not forget that the aim of cross-impact methods is to set forth the various implications of an expert's intuitions in a certain field. It would therefore be alarming to find something at the outcome the expert had not put in, or which would surprise him. On the other hand, when the problem has been qualitatively well-analysed beforehand, the many possible futures may be produced, and the assumptions required for long term forecasts may be accurately outlined using this method.

Appendix

Nonzero elements

The transition matrix of the process has as many lines and columns as there are states in the system, ie 2^n for n events. Therefore, there are $2^n \times 2^n = 2^{2n}$ elements. Among these, the probabilities corresponding to impossible transitions, ie a transition between a state in which an event has occurred to a state wherein it disappears, are zero.

The number of nonzero probabilities, starting from a state where p events have occurred, is equal to 2^{n-p}.

The number of states containing p events is equal to $C_n{}^p$, the number of nonzero elements is:

$$\sum_{p=0}^{n} C_n{}^p 2^{n-p} = (2+1)^n = 3^n$$

The transition probability

From assumption 2 (independence of events) all one has to do, to calculate the 2^{n-p} transition probabilities starting from a state containing p events, is to use the $n - p$ probabilities of the other events. The number of answers to be given is then equal to:

$$\sum_{n=0}^{n} (n - p)\, C_n{}^p$$

which can be calculated from the derivative of $y = (x + 1)^n$:

$$n(x + 1)^{n-1} = \sum_{p=0}^{n-1} (n - p)\, x^{n-p-1}\, C_n{}^p$$

Then, by putting $x = 1$, we obtain the result $n2^{n-1}$.

The transition probability to a state where events e_i occur and not events e_j is then equal to:

$$\Pi\ P(i)\ \Pi\ 1 - P(j),$$

where $P(i)$ is the probability of occurrence of event e_i.

Event possibilities

The number of paths of a process with 2^n states for m transitions is equal to 2^{nm}, ie approximately 10^{15} for five events in ten years; actually, several paths are impossible according to the definition we have given. The number of nonzero probability paths is only $(m + 1)^n$, since each event can occur either in one of the m transitions or after the last one: there are therefore $m + 1$ possibilities of occurrence for each event.

The time interval

The formula giving the new probabilities after reduction of time intervals is derived as follows: if we assume that the intervals given by the expert is broken down into k equal parts for which the probability of occurrence is unchanged, the probability that the event will occur during none of the k sub-intervals is

$$(1 - P')^k = 1 - P.$$

This formula is appropriate only if the events corresponding with the occurrence in each subinterval are independent, which is not exactly the case here since each event can only occur once. But we felt that it was more in agreement with the expert's intuition to consider that the probabilities he expresses correspond in fact to the accumulation of a set of circumstances in which the said event is unavoidable, and this set of circumstances can then happen several times during the period of time.

This procedure may be argued and replaced by $P' = P/k$ is preferred, but in any case this has but little impact on the sequence of events in a scenario.

The numbers of variables

One encounters difficulties when using a large number of variables. Obviously one cannot deal directly with a problem which consists of more than eight or nine events, even if they are represented in a condensed way in transitional matrices (so that only the nonzero elements are retained) or registered in a computer's fast peripheral memory.

This limitation is not due to the computer, but rather to the number of questions that have to be asked to allow the transitional probabilities to be calculated (about 5000 for 10 events).

The quality of the answers obtained in such an exercise would certainly be very mediocre, and the results would be of very little significance.

In fact there is no point in applying a cross-impact method to just any group of events, because to get a meaningful result, the participants must have a precise and equal understanding of each event and each state of the system in question. This implies that the expert whose opinion is asked and the user of the model would need to have a long discussion before giving their estimates of probabilities. It is therefore preferable to work with only five or six events.

If the problem studied is composed of a larger number than this, it can certainly be broken down into hierarchical subproblems by analysing the relationships between the events. When considering these relations the graph can simply be broken down into parts which are strongly connected and the priority ordering can thus be made apparent. Each of these parts constitutes a subsystem, which can be studied separately using the model, and has a limited number of variables. (The algorithm used in this process is simple and well known.)

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7. Several works present the theory of Markov chains, of which only the simpler results have been used here. See, for example A. Tortrat, *Processus aleatoire* (Paris, Dunod); Kai Lai Chung; *Markov Chains with Stationary Transition Probabilities* (Germany, Springer-Verlag).
8. F. Teniere-Buchot, "Le modèle POPOLE", *Analyse et prévision*, Sedeis (February/March 1973); J. C. Duperrin and M. Godet, "Prospective du systèmes de l'energie nucléaire", *Rapport économique CEA*, R 4541, December 1973.
9. This example was drawn up with the help of Mr Jean Claude Duperrin.